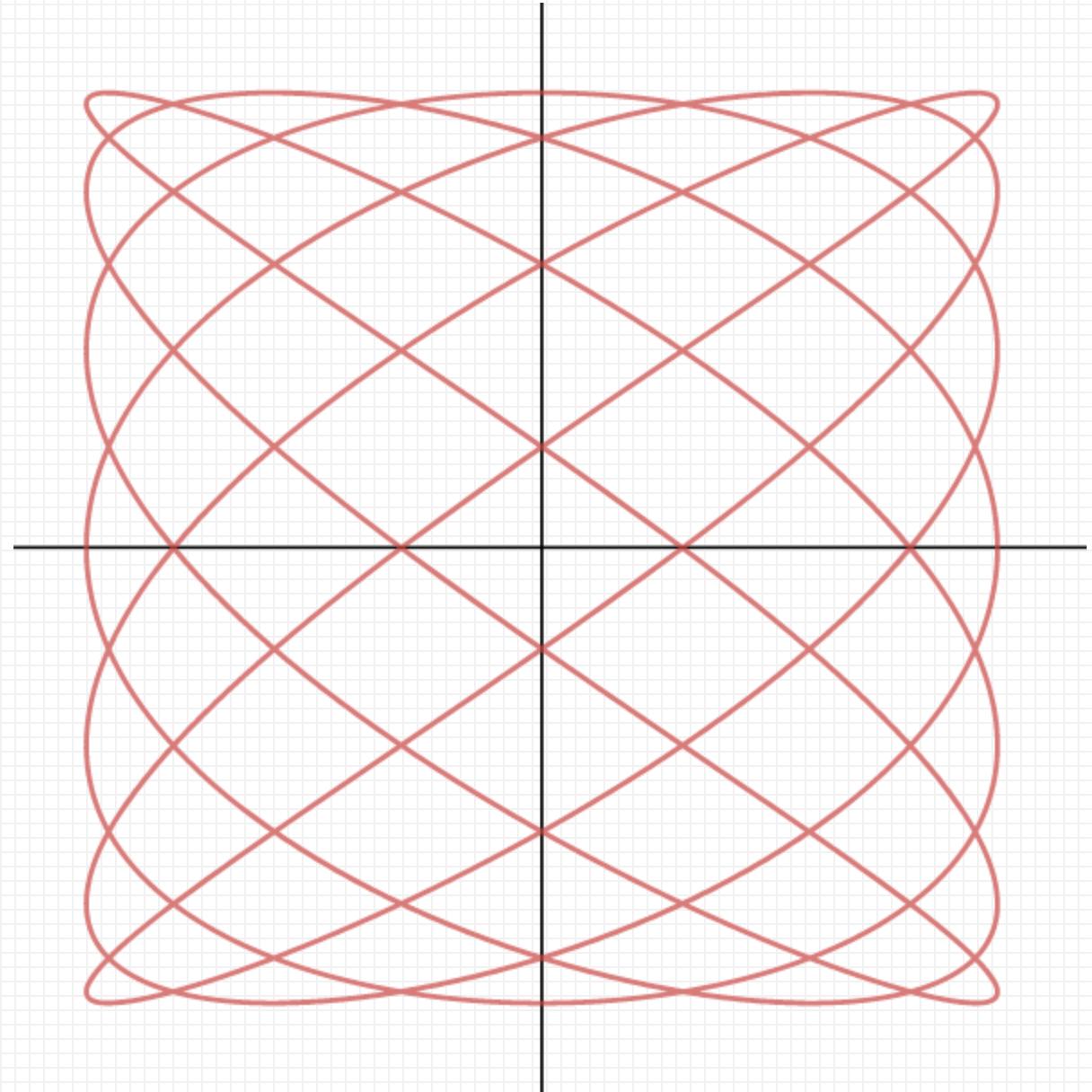
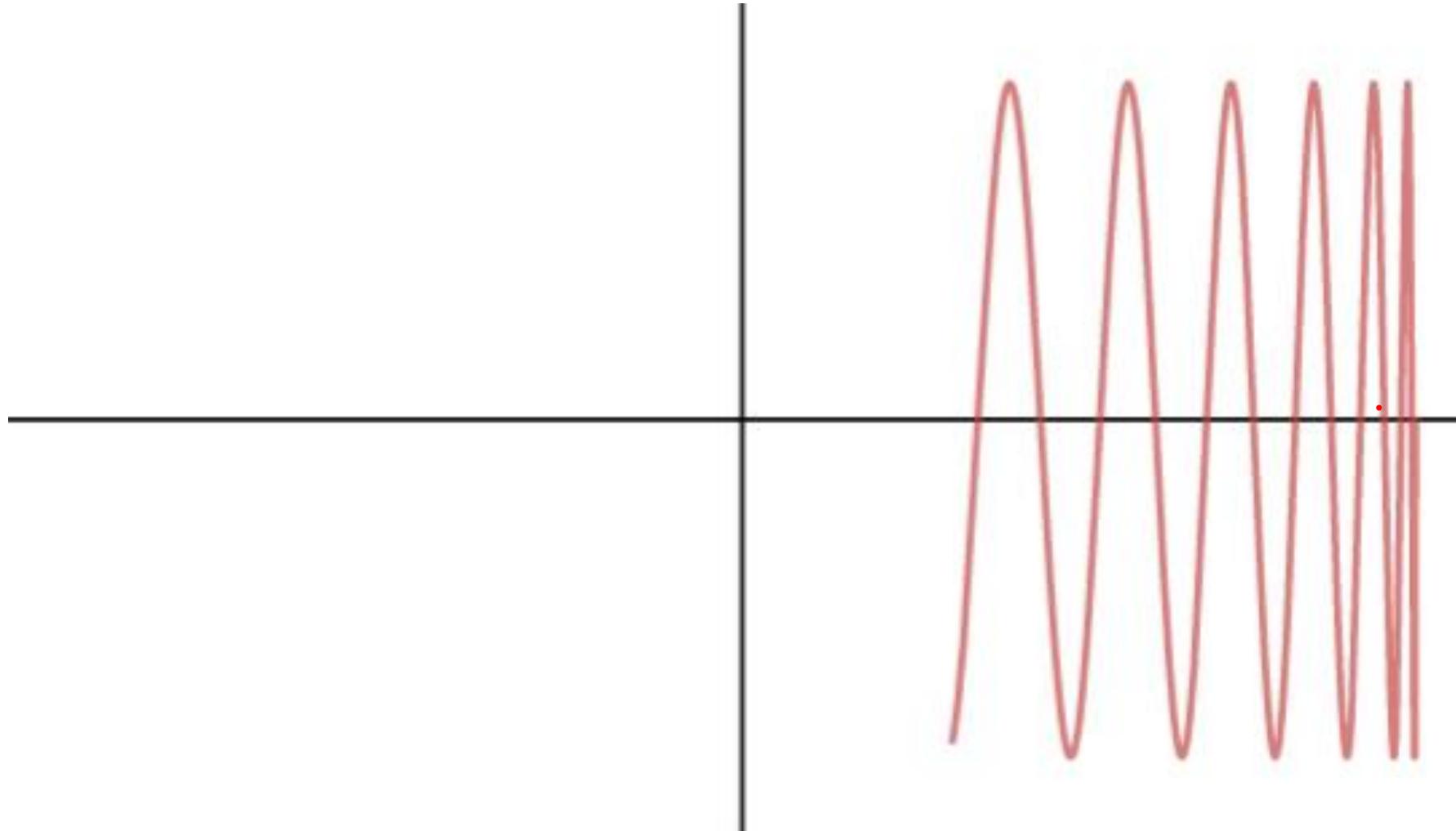


Section 10.1

Parametric Functions



Section 10.1 - Parametric Functions



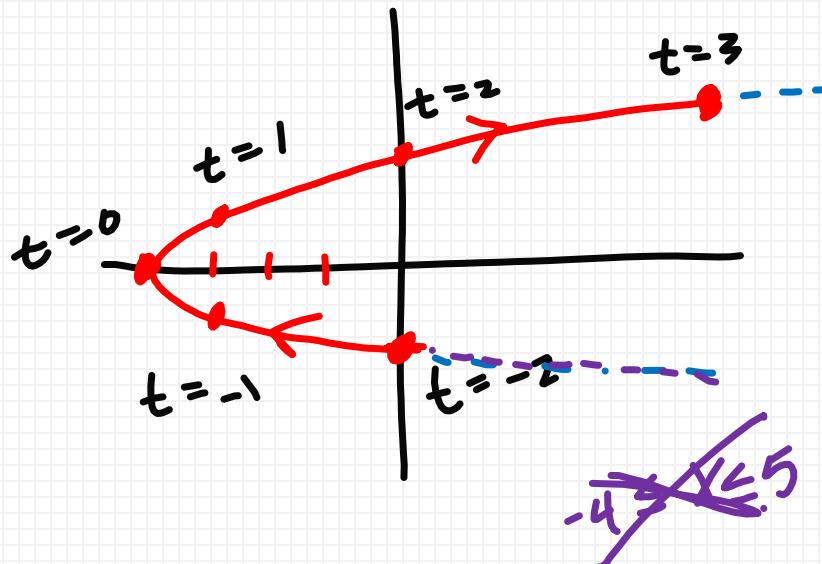
If x and y are given individually as functions $x = x(t)$, $y = y(t)$ over an interval of t , then the set of points $(x, y) = (x(t), y(t))$ is a parametric curve.

The equations are called parametric equations. The parameter is t .

Ex: $x = t^2 - 4$, $y = \frac{t}{2}$, $-2 \leq t \leq 3$

t	x	y
-2	0	-1
-1	-3	-1/2
0	-4	0
1	-3	1/2
2	0	1
3	5	3/2

Graph:



ELIMINATE THE PARAMETER

Rewrite in Cartesian ~~RECTANGULAR~~

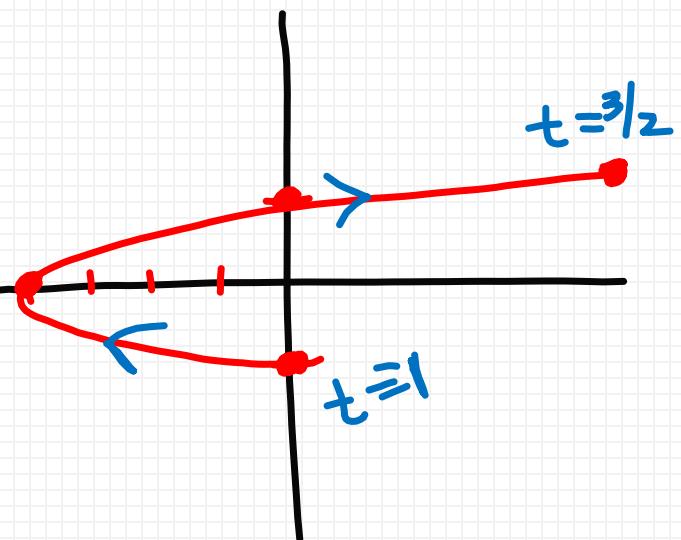
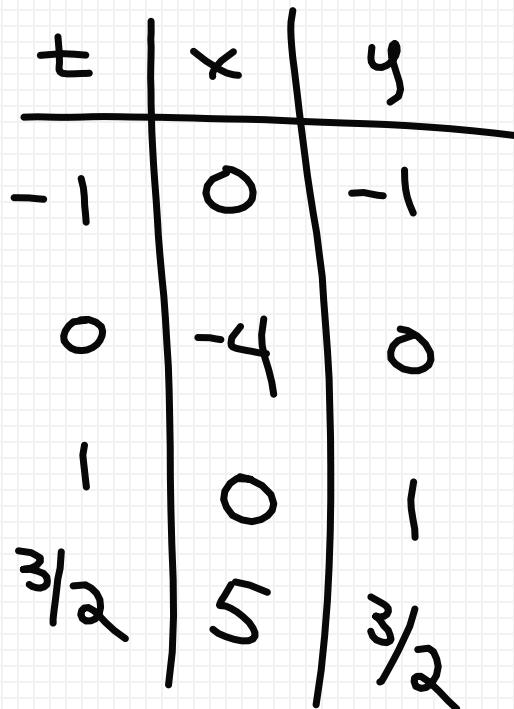
$$y = \frac{t}{2} \Rightarrow t = 2y$$

$$x = t^2 - 4 \Rightarrow x = (2y)^2 - 4$$

$$\boxed{x = 4y^2 - 4}, -1 \leq y \leq \frac{3}{2}$$



$$\text{Ex: } x = 4t^2 - 4, y = t, -1 \leq t \leq \frac{3}{2}$$

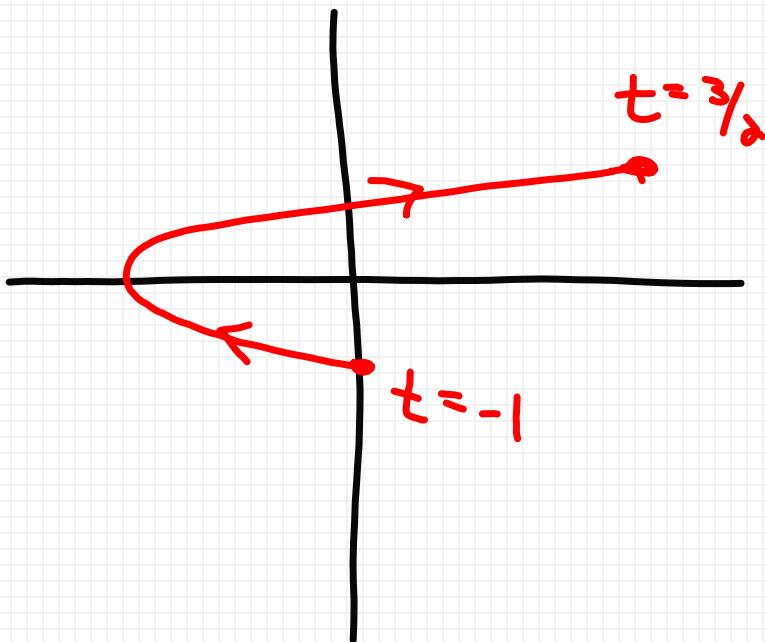


SAME GRAPH, BUT GRAPHS
"FASTER."
YOU CAN GET SAME GRAPH W/
DIFFERENT PARAMETRIC EQUATIONS.
PARAMETRIC EQ. ARE NOT UNIQUE.

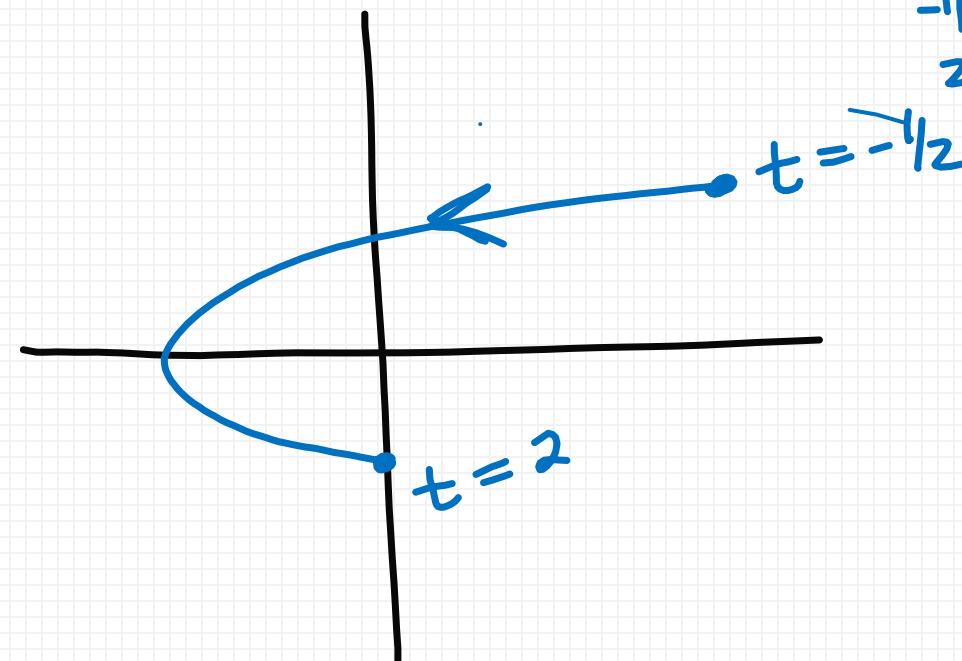


Use your calculator. Note any differences.

Ex: $x = 4t^2 - 4$, $y = t$, $-1 \leq t \leq \frac{3}{2}$



Ex: $x = 4t^2 - 8t$, $y = 1 - t$, $-\frac{1}{2} \leq t \leq 2$



t	x	y
$-\frac{1}{2}$	5	$\frac{3}{2}$
2	0	-1



Graph by eliminating the parameter first. Include a domain restriction if necessary.

$$x = \frac{1}{\sqrt{t+1}} \Rightarrow x \text{ must be } \underline{\underline{\text{positive}}}$$

$$x = \frac{1}{\sqrt{t+1}}, \quad y = \frac{t}{t+1}, \quad t \geq -1$$

$$\sqrt{t+1} = \frac{1}{x}$$

$$t+1 = \frac{1}{x^2}$$

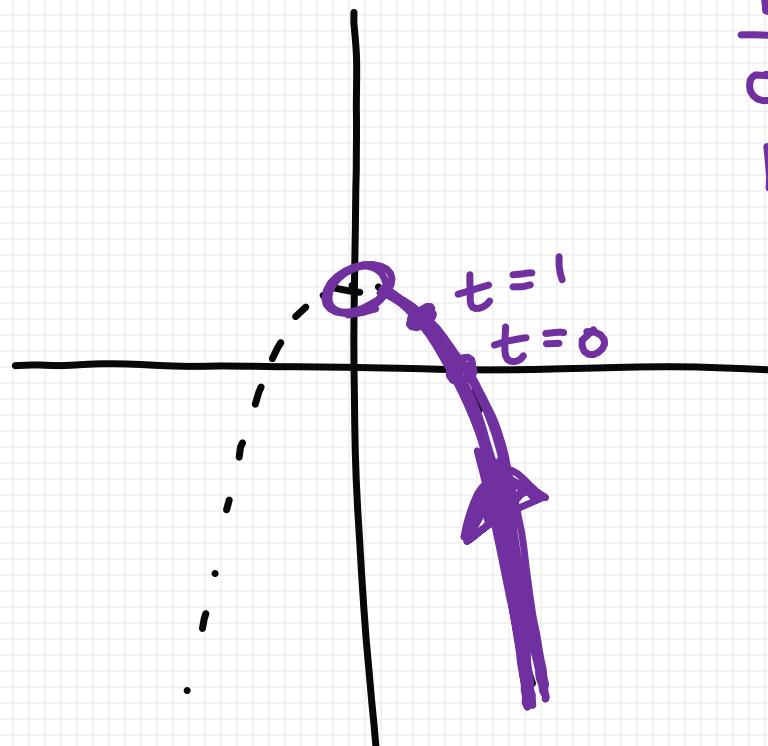
$$t = \frac{1}{x^2} - 1$$

$$y = \left(\frac{1}{x^2} - 1 \right) x^2$$

$$y = 1 - x^2$$

$$x \neq 0$$

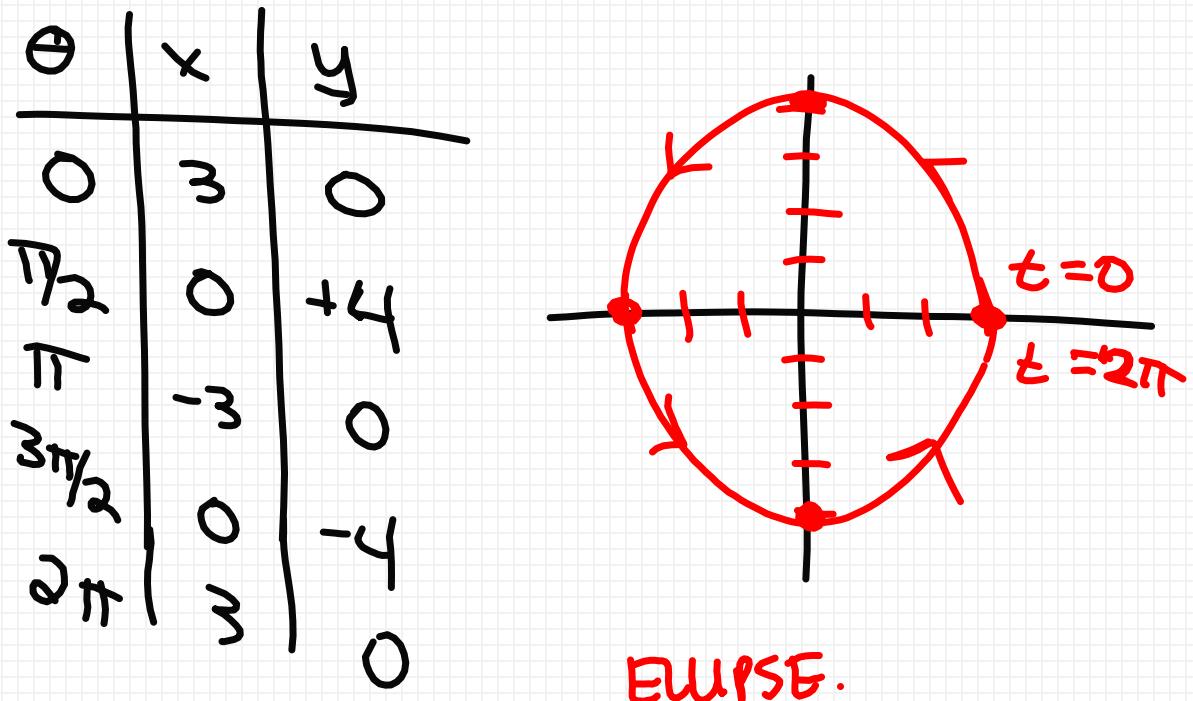
$$(x \neq 0)$$



t	x	y
0	1	0
1	1/2	1/2

Graph by eliminating the parameter first. Include a domain restriction if necessary.

$$x = 3\cos\theta, \quad y = 4\sin\theta, \quad 0 \leq \theta \leq 2\pi$$



$$x = 3\cos\theta \Rightarrow \frac{x}{3} = \cos\theta \Rightarrow \frac{x^2}{9} = \cos^2\theta$$

$$y = 4\sin\theta \Rightarrow \frac{y}{4} = \sin\theta \Rightarrow \underline{\frac{y^2}{16} = \sin^2\theta}$$

$$\frac{x^2}{9} + \frac{y^2}{16} = 1$$



**Practice Problems: Graph the following. Find the initial and terminal points.
Find the Cartesian equation.**

1. $x = 1 + \cos t, y = 3 - \sin t, [0, 2\pi]$

2. $x = t - 3, y = 3t - 7, [0, 3]$

3. $x = \sec t, y = \tan t, [\pi, 3\pi/2]$

4. $x = 4t + 3, y = 16t^2 - 9$

5. $x = t^2, y = 2 \ln t, [1, \infty)$



Answers:

1. $(x - 1)^2 + (y - 3)^2 = 1; x \in [0, 2]$

2. $y = 3x + 2; x \in [-3, 0]$

3. $x^2 - y^2 = 1; x \leq -1, y \geq 0$

4. $y = (x - 3)^2 - 9$

5. $y = \ln x; x \in [1, \infty)$

Homework:

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